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Research article

MATHEMATICAL MODEL TO MONITOR THE TRANSPORT OF DISSOLVED ZINC IN SEMI CONFINED AQUIFER INFLUENCED BY SEEPAGE VELOCITY IN OKIRIKA RIVERS STATE OF NIGERIA

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Abstract

Hydrogeological setting and formation characteristic has been the major influence on the quality of groundwater aquifer, formation characteristics are significant effect on the transport of solute such as dissolved zinc during transport process to groundwater aquifers, the study area were confirmed to have deposited semi confined aquifer. Such geological settings in coastal region are found to express various variations of geological formation in coastal aquifers. To monitor the transport of dissolved zinc in coastal aquifer, mathematical equation that govern the dissolved zinc were formulated through the variables expressed in the system, dissolved process of zinc in fresh water aquifers has a significant effect in the study area. Experts will find the developed model useful in the design of groundwater system to prevent dissolved zinc that deposit more than World Health Organization standard, contaminating ground water aquifer in the study area.

Keywords: mathematical model, dissolved zinc and seepage velocity

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1. Introduction

European river catchments are often densely populated and industrialized (Leuven and Poudevigne, 2002; Van der Velde et al., 2004). Due to a lack of water pollution control and wastewater treatment facilities, many river floodplains have become polluted in the past (Admiraal et al., 1993; Middelkoop, 1997; Albering et al., 1999; Vink et al., 1999b; Mertens et al., 2001). The effects of these pollutants on ecosystems are largely unknown, but local extinctions or population declines due to deteriorated river water, sediment and floodplain soil quality have been suggested in several studies (Balk et al., 1993; Kerkhofs et al., 1993; Hendriks et al., 1995; Kooistra et al., 2001). Deposited pollutants are subject to hydromorphodynamics (e.g. flooding, erosion and sedimentation processes) and turbation by animals. However, knowledge on the fate of heavy metals, especially in relation to turbation and inundation, is scarce of Floodplain soils often abundantly harbour burrowing animals, so-called bioturbators, including various mammals (e.g. voles, mice and moles) and soil macro-invertebrates, like earthworms and insects and their larvae (Mitchell, 1988; Mu¨ller-Lemans, 1996; Tyler et al., 2001). Bioturbation processes include digging, casting, and construction of nests and burrows. Bioturbation occurs especially in the upper 20 cm of soils, where the most recently deposited pollutants are present (Middelkoop, 1997). Some species like epigeic earthworms (e.g. Lumbricus rubellus) are especially active in the upper 3 cm of topsoil (Zorn, 2004), and all species burrowing deeper but frequently surfacing (e.g. endogeic and anecic earthworm species and underground dwelling small mammal species) or species burrowing from the surface to deeper layers (e.g. rabbits and voles searching for food) turbate the topsoil as well. Zinc is a widespread heavy metal in river systems, occurring in elevated and potentially toxic quantities all over Europe (Balk et al., 1993; Kalbitz and Wennrich, 1998).

Zinc is essential for life in all organisms but is toxic in excess, which requires homeostatic mechanisms to control intracellular zinc levels. E\ux of zinc in Escherichia coli is accomplished by the P-type ATPase ZntA and the cation di!usion facilitator (CDF) ZitB The CDF family Grass et 2001,Njes and Silven, 1996 stated that of proteins has common structural characteristics, with (in most cases) six transmembrane helices and N- and C-terminal histidine-rich motifs predicted to extend into the cytosol. These membrane transporters are usually involved in zinc transport across cytoplasmic or organelle membranes [3^6]. Some prokaryotic CDF proteins also transport cobalt and cadmium [7^10]. Recently, Gu!anti et al 2002. Rosen el at 1978, Padan and Schaldener, 1994 showed that CzcD from Bacillus subtilis utilizes an antiport mechanism. Antiporters are secondary transporters that couple electrochemical gradients of ions or organic solutes to drive transport reactions Rosen el at 1978, Padan and Schaldener, 1994. CzcD catalyzes active e\ux of Zn2þ in exchange for Kþ and Hþ [10]. However, the amino acid residues that participate in catalysis are unknown. Secondary active transport proteins convert free energy stored in electrochemical ion gradients into work in the form of a concentration gradient. Comprehensively studied examples include the proton/substrate symporter lactose permease (LacY) and the Naþ/Hþ Antiporters NhaA and NhaB kaback and Rashet 1978. Surprisingly, extensive use of site-directed mutagenesis demonstrated that only six amino acid residues in LacY are irreplaceable with respect to active lactose transport. Charge pairs have been identi ¢ ed that mediate substrate binding and Hþ translocation.

2. Theoretical background

Environmental pollution is a serious threat to Deltaic environment, the studies area, is prone to high percentage predominant of higher degree of porous medium in the geological formations, as investigated by various experts. Different sources of pollution are found in the study location, an average percentage of the pollution is generated from manmade activities. These includes, waste from industries and other sources. Other sources of this pollution are natural origin, these are the saline intrusion from the coastal environment, this implies that the study location has sources of pollution from the man made activities and natural origin, but the focus of this study is from manmade activities. The source of dissolved zinc in the study location is from the activities of man. Dissolved zinc are trace from ground water aquifer, which were found to be a serious threat to auriferous zone and water quality, the reason is that there are several regeneration of this solute almost every day, polluting the soil leaching to ground water aquifer. These sources of environmental influence generate the solute from the industries and other sources of man activities. This condition has seriously and rapidly decreased the standards quality of coastal aquifer deposited in the study location, based on these ugly scourge, there is need to take action to manage and prevent these sources of pollution from the activities of man, this is to develop a conceptual framework to manage the natural origin in the study area. To manage the pollution source and prevent the solute transport in coastal aquifer, mathematical equations were formulated by considering the variables that influence the transport of dissolved zinc in the study area. The study was carried out to monitor the transport between the lateritic and silty sand formations, the reason of considering these formations is based on the geological formation that were confirmed to deposit. Shallow short fresh water aquifer, this expressed the aquiferous zone in coastal aquifer, this resulted from the development of mathematical model will monitor the lithology of the study area. And were the concentrations are within the limit of ground water quality. The governing equations to monitor the transport of this dissolved zinc are expressed below.

3. Governing Equation

$$\frac{Vi\,\partial C}{\partial xi} = \frac{qi}{\varepsilon} = \frac{-Kih}{\varepsilon}\,\frac{\partial C}{\partial xi} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$\frac{\partial C}{\partial x} = SC_{(x)} - C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

$$\frac{\partial C}{\partial xi} = SC_{(x)} - C_0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$C = C_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (4)$$

Substituting equations (2), (3) and (4) into equation (1) yield

$$Vi\left[SC_{(x)} - SC_{(x)} - C_{(0)}\right] - \frac{qi}{\varepsilon} - \frac{Kih}{\varepsilon}\left[SC_{(x)} - C_{(x)}\right] - C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5)$$

$$Vi\ SC_{(t)} - ViSC^{1}{}_{(t)} - C_{(0)} - \frac{qi}{\varepsilon}\,\frac{Kih}{\varepsilon}SC_{(0)} + \frac{qi}{\varepsilon}\,\frac{Kih}{\varepsilon}C_{(0)} - C_{(0)} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

Equations (1) are the formulated governing equation that expressed the transport of dissolved zinc in lateritic and silty formations in the study area.

The expressions were derived by applying Laplace transformation to express the subject reaction of the variables in terms of expressing their functions on the transport process of initial concentration.

This expression from equation (1) to (4) were transformed and substituted to yield equation (5) and (6).

Considering the following boundary condition at $t = 0, C^1_{(0)} = C_0 = 0$ …… (7)

We have

Boundary conditions were expressed in equation (7) where the boundary values were expressed as t = 0 $C_{(0)}$ = Co = 0.

$$C_{(x)}\left(ViS - Vs - \frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}S\right) = 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (8)$$

$$C_{(t)} \neq 0 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (9)$$

But considering the boundary condition

At $t > 0, C^1_{(0)} = C_{(0)} = C_o$ ……………………………. (10)

$$SC_{(x)} - \frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}S_{(x)} - ViSc_o + ViC_o + \frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (11)$$

Applying the expression, with respect to the concentration on distance developed equation (8) where the expression of concentration with respect to time on the transport process were expressed as $C_{(t)} \neq 0$ at equation (9).

Take the time through the boundary values at equation (10) it developed the derived expression of equation (11) where concentration under the influence of seepage velocity were and aquifer thickness were expressed, this is to monitor their functional relation on the transport process of the concentration at various formations

$$\left[ViS - \frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}S\right]C_{(x)} = \left[ViS + Vi + \frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}\right]C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (12)$$

$$\frac{C_{(x)} = ViS + Vi\,\frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}}{ViS - \frac{qi}{\varepsilon}\frac{Kih}{\varepsilon}S}C_o \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (13)$$

Applying quadratic expression, we have

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \qquad (14)$$

The derivation of these expressions continued so that other parameters will express the functionality in the system as it is expressed in equation (13).

The subjection relation on equation (13) were converted for thorough explicit of the variables by applying quadratic expression.

The parameters were integrated in phase based on variation of the formation thus the behaviour of the formation characteristics that influence the system. Subject to this application, the parameter considered in the system are functional in accordance with the transport behaviour of the concentration influenced by seepage velocity in the system. This condition were subjected to the influence from porosity, seepage velocity, aquifer thickness, time of dissolved zinc and distance travel from one region to the other. This expressed were expressed during the process of parameter expressing there functions with one another with respect to time of concentration and distance travelled from equation (14)

to Where a = Vi, b = $\frac{qi}{\varepsilon}$, c = $\frac{Kih}{\varepsilon}$

$$S = \frac{\frac{qi}{\varepsilon} - \sqrt{\frac{qi}{\varepsilon}^2 + 4Vi\frac{Kih}{\varepsilon}}}{2Vi} \qquad \text{.................................... (15)}$$

$$S_1 = \frac{\frac{qi}{\varepsilon} - \sqrt{\frac{qi}{\varepsilon}^2 - 4Vi\frac{Kih}{\varepsilon}}}{2Vi} \qquad \text{.................................... (16)}$$

$$S_2 = \frac{\frac{qi}{\varepsilon} + \left[\sqrt{\frac{qi}{\varepsilon}^2 + 4Vi\frac{Kih}{\varepsilon}}\right]}{2Vi} \qquad \text{.................................... (17)}$$

$$S_1 = \frac{\frac{qi}{\varepsilon} + \left[\sqrt{\frac{qi}{\varepsilon}^2 + 4Vi\frac{Kih}{\varepsilon}}\right]}{2Vi} S_2 + \frac{\frac{qi}{\varepsilon} - \left[\sqrt{\frac{qi}{\varepsilon}^2 + 4Vi\frac{Kih}{\varepsilon}}\right]}{2Vi} \ell^{\frac{\left[\frac{qi}{\varepsilon} + \sqrt{\frac{qi}{\varepsilon}^2 + 4Vi\frac{Kih}{\varepsilon}}\right]^{\frac{L}{v}}}{2Vi}} +$$

$$\frac{\left[-\frac{qi}{\varepsilon} - \frac{qi}{\varepsilon}\sqrt{\frac{qi}{\varepsilon}^2 + 4Vi\frac{Kih}{\varepsilon}}\right]}{2Vi} \qquad \text{.................................... (18)}$$

Applying inverse of the equation, we obtain

$$C_{(x)} = \left[\frac{Vi}{x} + Vi + \frac{qi}{\varepsilon}\right] C_o \, \ell^{\frac{\left[\frac{qi}{\varepsilon} + \sqrt{\frac{qi}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}}\right]^x}{2Vi}} + \ell^{\frac{\left[\frac{qi}{\varepsilon} - \sqrt{\frac{qi}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}}\right]^x}{2Vi}} \qquad \text{.................. (19)}$$

But if $x = \frac{t}{v}$

$$\left[C\,[t,v] = {Vi}\big/{t/v} + Vi + \frac{qi}{\varepsilon} \right] C_o \ell^{\frac{\left[\frac{qi}{\varepsilon} + \sqrt{\frac{qi^2}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}} \right]^{\frac{t}{v}}}{2Vi}} \qquad \text{..................} \quad (20)$$

Considering the following boundary condition at

$$t = 0, C^1{}_0 = 0, C_0 = 0 \qquad \text{..............................} \quad (21)$$

$$C_{(x)} = \left[\frac{Vi}{x} + Vi + \frac{qi}{\varepsilon} \right] C_o \ \ell^{\frac{\left[\frac{qi}{\varepsilon} + \sqrt{\frac{qi^2}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}} \right]^{\frac{t}{v}}}{2Vi} + \frac{\left[\frac{qi}{\varepsilon} \sqrt{\frac{qi^2}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}} \right]^{\frac{t}{v}}}{2Vi}} \qquad \text{............................} \quad (22)$$

At $C^1{}_{(0)} = t \neq 0$

Again $C_{(0)} = C_{(0)}$

So that $C_O = \left[Vi + \frac{qi}{\varepsilon} \right] C_o \,[1 + 1]\, i.e.\ 0 = \left[0 + \frac{qi}{\varepsilon} \right]$ .... (23)

$$\Rightarrow \qquad \frac{qi}{\varepsilon} + \frac{qi}{\varepsilon} = 0 \qquad \text{............................} \quad (24)$$

So that we have

$$C_{(x)} = \left[2\frac{Vi}{x} \right] C_o \ \ell^{\frac{\left[\frac{qi}{\varepsilon} \sqrt{\frac{qi^2}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}} \right]^{\frac{t}{v}}}{2Vi} + \frac{\left[\frac{qi}{\varepsilon} - \sqrt{\frac{qi^2}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}} \right]^{\frac{t}{v}}}{2Vi}} \qquad \text{............................} \quad (25)$$

However, $e^x + e^{-x} = 2Cos\,x$ therefore, we have

$$\boxed{C_{(x)} \left[2\frac{Vi}{t} \right] C_o \ Cos \ \frac{\left[\frac{qi}{\varepsilon} \sqrt{\frac{qi}{\varepsilon} + 4Vi\frac{Kih}{\varepsilon}} \right]^{\frac{t}{v}}}{2Vi}} \qquad \text{............................} \quad (26)$$

Equation (26), where the final expressed mathematical model, this derived model will monitor the transport of dissolved zinc influenced by the seepage velocity in coastal area of Okirika. The model expressed in equation (26) accommodated the parameters distributed in mathematical tools; it is expressed in a condition that will develop theoretical solutions that will determine the rate of dissolved zinc in coastal aquifer.

4. Conclusion

Monitoring the transport of dissolved zinc in semi confined aquifers influenced by seepage velocity in coastal area of Okirika has been mathematically expressed through the derived mathematical model. The developed will monitor the rate of contaminant at various formations.

There are lots of influences on the transport of the dissolved zinc in the coastal aquifer. These conditions were expressed through the study variables degree of knowledge and its management practice, ranging from conceptual model about aquifer behaviour and comprehensive management action, such as aquifer vulnerability mapping, because there are variability of hydrogeologic setting, including the distribution of zinc, saline water and history of groundwater withdrawals and fresh water drainage that has resulted in a variety of models of zinc and saltwater intrusion in coastal aquifer system. Such information was varieties and other influences from the formation characteristics in the coastal area of Okirika. When semi confined bed were confirmed to deposit, in the study location. The influence of seepage velocity were confirmed through the geological setting in the study area, these variables where applied to formulate the governing equation, the developed model from the governing equation will monitor the transport of dissolved zinc in semi confined bed influence by seepage velocity in the study area.

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